Categorising quadratics by the remainder theorem

Think about all quadratic expressions and the three properties:

A: The remainder is 1 when divided by $x+1$

B: $x-1$ is a factor of the expression

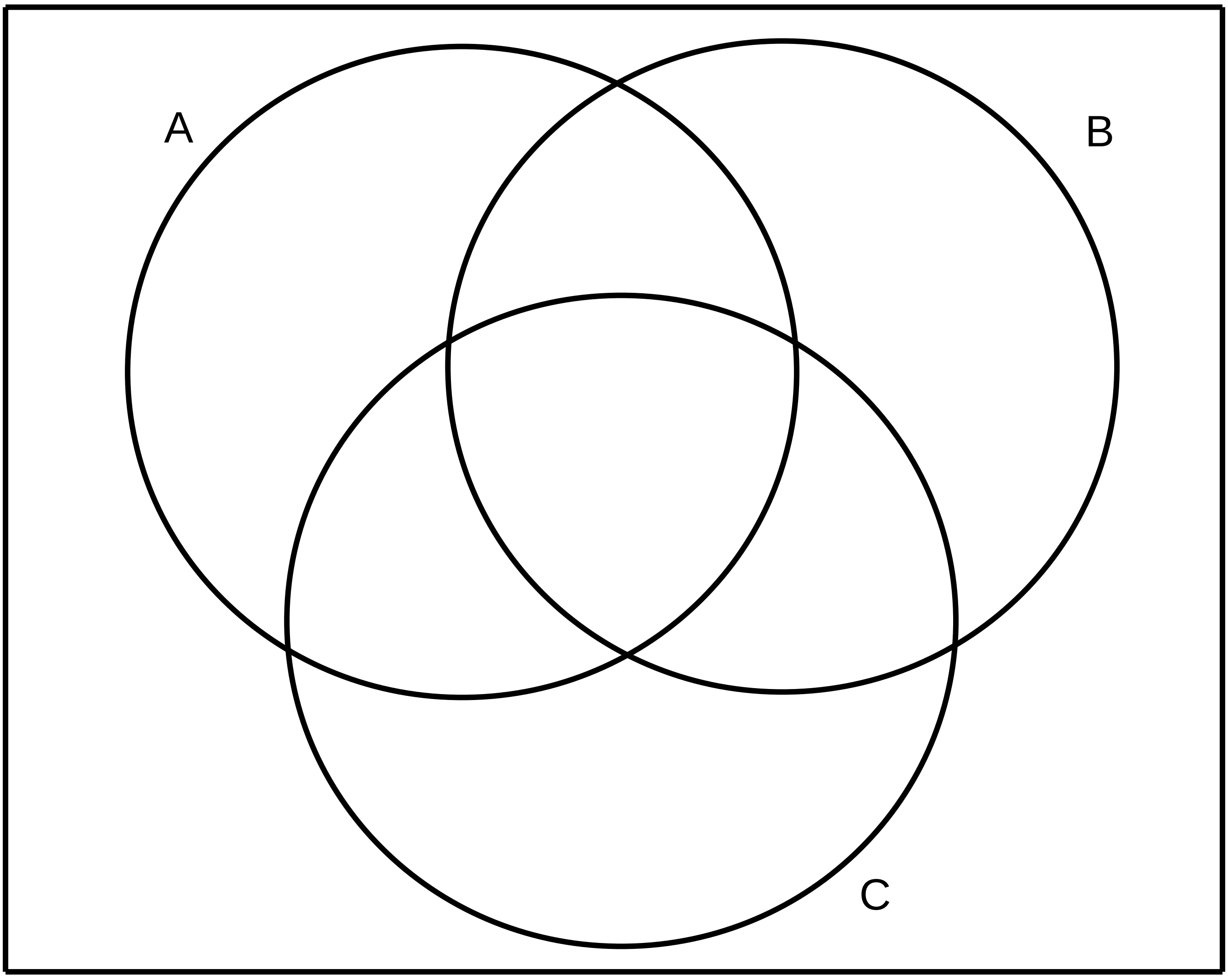
C: $x-2$ is a factor of the expression

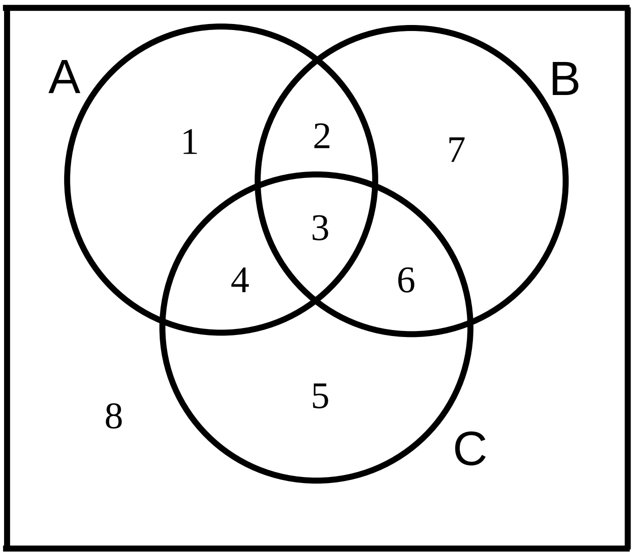
Can you find a quadratic expression which satisfies all three properties A, B and C? If so write this quadratic in the central region where the three circles overlap.

How about a quadratic expression which doesn't satisfy any of the requirements? Fill this in the region outside the three circles.

The task: To find one example for each of the other six regions.

Is it possible to find an example for every region?





A possible solution

A: The remainder is 1 when divided by $x+1$

B: $x-1$ is a factor of the expression

C: $x-2$ is a factor of the expression

1 $\mathrm{f}(x)=\frac{1}{6}(13x^2-9x-16)$	2 $\mathrm{f}(x)=\frac{1}{6}(7x^2-3x-4)$
3 $\mathrm{f}(x)=\frac{1}{6}(x^2-3x+2)$	4 $\mathrm{f}(x)=\frac{1}{6}(7x^2-9x-10)$
5 $\mathrm{f}(x)=x^2-x-2$	6 $\mathrm{f}(x)=x^2-3x+2$
7 $\mathrm{f}(x)=x^2-1$	8 $\mathrm{f}(x)=3x^2-4x-1$

How did we find these?
Think about $\mathrm{f}(x)=a(x-1)(x-2)+b(x-1)(x+1)+c(x-2)(x+1)$ where a,b,c and R are constants.

a	b	c	$\mathrm{f}(x)$	Region
0	0	1	$\mathrm{f}(x)=(x-2)(x+1)$	5
$\frac{1}{6}$	0	1	$\mathrm{f}(x)=(x-2)\left(\frac{1}{6}(x-1)+(x+1)\right)$	4
0	1	0	$\mathrm{f}(x)=(x-1)(x+1)$	7
$\frac{1}{6}$	1	0	$\mathrm{f}(x)=(x-1)\left(\frac{1}{6}(x-2)+(x+1)\right)$	2
1	0	0	$\mathrm{f}(x)=(x-1)(x-2)$	6
$\frac{1}{6}$	0	0	$\mathrm{f}(x)=\frac{1}{6}(x-1)(x-2)$	3
$\frac{1}{6}$	1	1	$\mathrm{f}(x)=\frac{1}{6}(x-1)(x-2)+(x-1)(x+1)+(x-2)(x+1)$	1
1	1	1	$\mathrm{f}(x)=(x-1)(x-2)+(x-1)(x+1)+(x-2)(x+1)$	8

Although this doesn't follow the generating pattern used, for region 1 there is an easier approach: $\mathrm{f}(x)=(x+1)^2+1$ and for region 8: $\mathrm{f}(x)=2x^2$ but not $\mathrm{f}(x)=x^2$ (Why?)

An easier problem:
Give your students the eight quadratics to place in the Venn diagram.