Edexcel Core 2 Geometry and trigonometry

Section 1: Coordinate geometry of circles

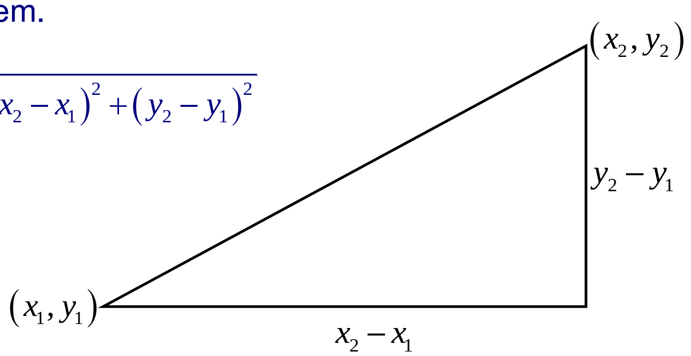
Notes and Examples

These notes and examples contain subsections on

The mid-point of a line

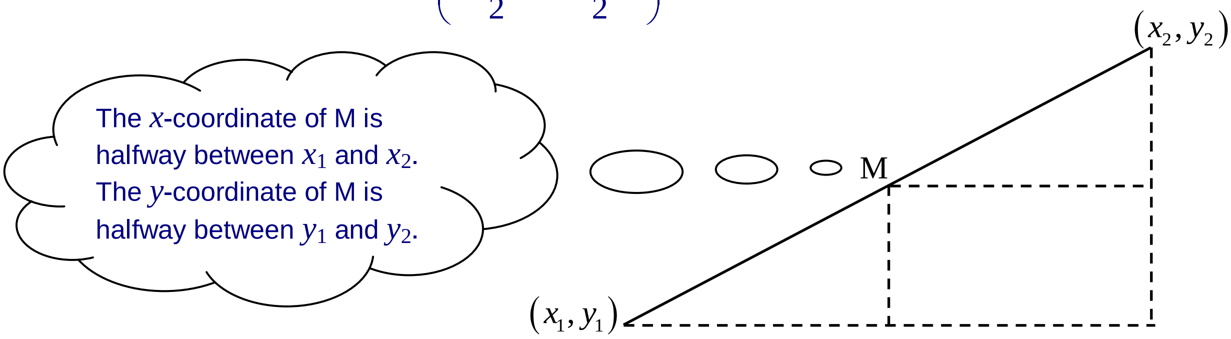
The length of a line joining two points (x_1, y_1) and (x_2, y_2) can be found using Pythagoras' Theorem.

Length $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

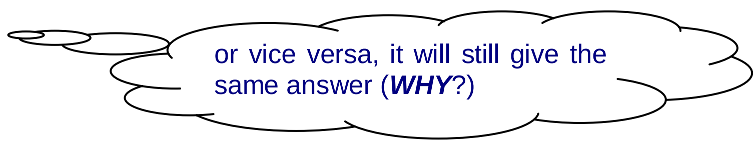


The midpoint of a line joining two points (x_1, y_1) and (x_2, y_2) is given by

Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



Example 1
A is the point (2, -6). B is the point (-3, 4).
Calculate
(i) the midpoint of AB
(ii) the length of AB.
Solution



Choose A as (x_1, y_1) and B as (x_2, y_2).

(i) Midpoint is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

i.e. $\left(\frac{2+(-3)}{2}, \frac{-6+4}{2}\right)$

$= \left(\frac{-1}{2}, -1\right)$

(ii) The distance AB is given by

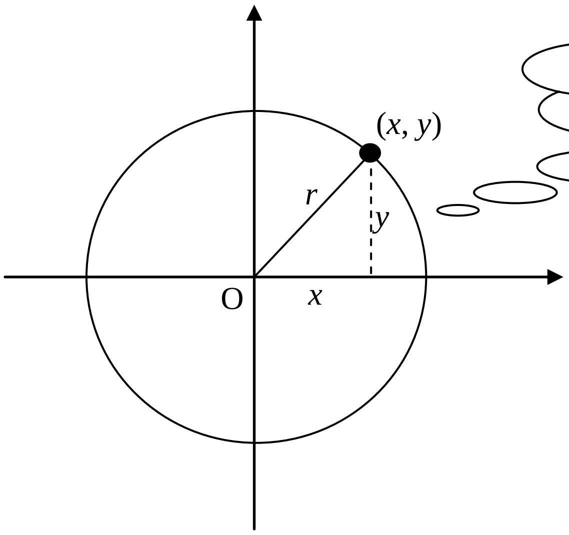
$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(2-(-3))^2 + ((-6)-4)^2}$$
$$= \sqrt{(5)^2 + (-10)^2}$$
$$= \sqrt{25+100}$$
$$= \sqrt{125}$$

Note: The answer is often left like this if the square root is not exact. However since 125 = 25×5 then $\sqrt{125} = \sqrt{25}\sqrt{5} = 5\sqrt{5}$ is perhaps a simpler form.

The equation of a circle

Start this section by looking at either the Geogebra resource ***Circles*** or the ***Circles dynamic spreadsheet*** (selecting the *Circle Equations* sheet). First, set the centre of the circle to be the origin and vary the radius. Look at how the equation of the circle changes.

A circle with centre the origin and radius r is the locus of all points whose distance from the origin is r units.



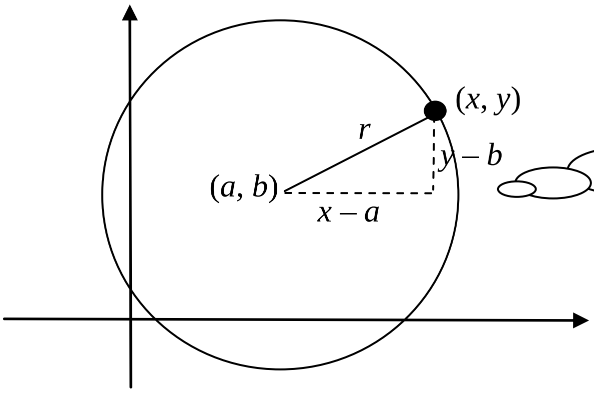
For all points (x, y) on the circumference of the circle, $x^2 + y^2 = r^2$ by Pythagoras' theorem.

The general equation of a circle, centre (0, 0) and radius r is $x^2 + y^2 = r^2$.

Edexcel C2 Geometry & Trig 1 Notes & Examples

Now use the Circles dynamic spreadsheet or the Geogebra Circles resource again. This time vary the coordinates of the centre of the circle, and look at how the equation of the circle changes.

A circle with centre (a, b) and radius r is the locus of all points whose distance from the point (a, b) is r units.



For all points (x, y) on the circumference of the circle, $(x-a)^2+(y-b)^2=r^2$ by Pythagoras' theorem.

The general equation of a circle, centre (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$.

Example 2

For each of the following circles find (i) the coordinates of the centre and (ii) the radius.

(a) $x^2 + y^2 = 49$

(b) $(x + 2)^2 + (y - 6)^2 = 9$

Solution

(a) $x^2 + y^2 = 49$ can be written as $x^2 + y^2 = 7^2$.

(i) The coordinates of the centre are (0, 0)

(ii) The radius is 7.

This is a particular case of the general form $x^2 + y^2 = r^2$ which has centre (0, 0) and radius r.

(b) $(x + 2)^2 + (y - 6)^2 = 9$ can be written as $(x - (-2))^2 + (y - 6)^2 = 3^2$.

(i) The coordinates of the centre are (–2, 6)

(ii) The radius is 3.

This is a particular case of the general form $(x - a)^2 + (y - b)^2 = r^2$ which has centre (a, b) and radius r.

Sometimes the circle equation needs to be rearranged into its standard form before you can find the centre and radius. To do this, you need to complete the square on the x terms and on the y terms.

Example 3

Show that the equation $x^2+y^2+4x-6y-3=0$ represents a circle, and find its centre and radius.

Solution

$x^2 + y^2 + 4x - 6y - 3 = 0$

$x^2 + 4y + y^2 - 6y = 3$

$(x+2)^2 - 4 + (y-3)^2 - 9 = 3$

$(x+2)^2 + (y-3)^2 = 16$

This is the equation of a circle, centre (-2, 3), radius 4.

Alternatively, you can use the approach in the book involving the general equation of the circle (see Example 13.1.6 on page 199).

Try the ***Circles walkthrough*** and ***Equations of circles skill pack***.

You may find the Mathcentre video ***Coordinate geometry of a circle*** useful.

Finding the equation of a circle

In Section 1 you looked at different ways of finding the equation of a line. You can find the equation of a line from the gradient and the intercept, or from the gradient and one point on the line, or from two points on the line.

In the same way, there are several ways of finding the equation of a circle, depending on the information available.

Finding the equation of a circle from the radius and centre

Example 4

Find the equation of each of the following.

(a) a circle, centre (0, 0) and radius 4.

(b) a circle, centre (3, –4) and radius 6.

Solution

(a) The equation of a circle centre the origin is $x^2 + y^2 = r^2$

$r = 4$ so the equation is $\quad x^2 + y^2 = 4^2$

i.e. $\quad x^2 + y^2 = 16$

(b) The equation of a circle centre (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$

$a = 3$, $b = -4$ and $r = 6$ so the equation is $\quad (x - 3)^2 + (y - (-4))^2 = 6^2$

i.e. $\quad (x-3)^2 + (y+4)^2 = 36$

Edexcel C2 Geometry & Trig 1 Notes & Examples

Finding the equation of a circle from its centre and one point on its circumference

If you know the centre of the circle and one point on its circumference, you can find the radius by calculating the distance between these two points. You can then find the equation of the circle.

Example 5
Find the equation of the circle, centre (1, -2), which passes through the point (-2, -3).

Solution
The distance *r* between (1, -2) and (-2, -3) is given by:

$$r = \sqrt{(1-(-2))^2 + (-2-(-3))^2}$$
$$= \sqrt{3^2 + 1^2}$$
$$= \sqrt{10}$$

The radius of the circle is therefore $\sqrt{10}$.
The equation of the circle is $(x-1)^2 + (y+2)^2 = 10$

For practice in examples like the one above, try the interactive resource ***Find the equation of a circle***.

Circle geometry

There are three important facts about circles that you need to know. These facts often help to solve problems involving circles.

1. The angle in a semicircle is a right angle.
2. The perpendicular from the centre of a circle to a chord bisects the chord.
3. The tangent to a circle is perpendicular to the radius at that point

You can see demonstrations of these properties using the GeoGebra Explore resource ***Circle properties***.

Keep these properties in mind when dealing with problems involving circles. Sometimes using these properties can make solving problems very easy!

You should draw a sketch diagram when solving problems involving coordinate geometry.

For some practice in using the third property, try the ***The tangent to a circle skill pack***.

The intersection of a line and a circle

Look at the ***Circles dynamic spreadsheet***. Select the sheet *Circle and a line*. Try varying the equation of the line and/or the circle, and make sure that you can see that there may be two intersections, no intersections or one intersection (in which case the line touches the circle).

You can also try the ***Circle and line intersection skill pack***.

Example 6

Find the coordinates of the point(s) where the circle $(x+2)^2+(y-1)^2=9$ meets

(i) the line $y = 5$

(ii) the line $x = 1$

(iii) the line $y = 2 - x$

Solution

(i) Substituting $y = 5$ into the equation of the circle:

$$(x+2)^2+(5-1)^2=9$$
$$(x+2)^2+16=9$$
$$(x+2)^2=-7$$

The expression $(x + 2)^2$ cannot be negative

There are no solutions. The line does not meet the circle.

(ii) Substituting $x = 1$ into the equation of the circle:

$$(1+2)^2+(y-1)^2=9$$
$$9+(y-1)^2=9$$
$$(y-1)^2=0$$
$$y=1$$

The point is on the line $x = 1$, so its x-coordinate must be 1.

The line touches the circle at (1, 1).

(iii) Substituting $y = 2 - x$ into the equation of the circle:

$$(x+2)^2+(2-x-1)^2=9$$
$$(x+2)^2+(1-x)^2=9$$
$$x^2+4x+4+1-2x+x^2=9$$
$$2x^2+2x-4=0$$
$$x^2+x-2=0$$
$$(x-1)(x+2)=0$$
$$x=1 \text{ or } x=-2$$

When $x = 1$, $y = 2 - 1 = 1$

When $x = -2$, $y = 2 - (-2) = 4$

Substitute the x values into the equation of the line to find the y-coordinates.

The line crosses the circle at (1, 1) and (–2, 4).